



# Experimental validation of variance capture in sample splitting for heterogeneity tests

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## Abstract

Accurate particulate analysis in industries such as mining, agriculture and pharmaceuticals depends critically on representative sample division. While rotary splitters are widely recognized for their practical efficiency and representativeness, their systematic nature is theoretically incompatible with the randomness required for calibrating Gy's Fundamental Sampling Error formula in heterogeneity studies. This paper experimentally investigates whether a rotary splitter suppresses compositional variance compared to the conventional riffle splitter when operated at different split ratios. Using a seeded basalt lot with tracer additives of varying densities, the performance of a Rocklabs Boyd Elite Rotary Sample Divider was evaluated at split ratios from 5% to 25% and benchmarked against both a vibratory feeder (worst-case heterogeneity) and a 20-vane riffle splitter. Results demonstrate that the rotary splitter significantly suppresses variance at higher split ratios ( $\geq 15\%$ ), reducing particle count variability by 40–50% compared to the riffle splitter. This variance suppression is attributed to the larger sample mass and the high number of incremental cuts ( $\sim 246$  vs. 20), which more effectively average out local segregation. The study concludes that while rotary splitting is unsuitable for Fundamental Sampling Error calibration due to its non-random mechanism, it is a superior tool for routine sample preparation where compositional representativeness is the primary goal, provided it is operated at an appropriate split ratio.

## Keywords

Rotary splitting, riffle splitting, variance suppression, sample heterogeneity, Gy's formula, particulate sampling

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## Introduction

Representative division of particulate materials is a cornerstone of reliable analysis in industries reliant on particulate characterization, including mineral processing, agriculture and pharmaceuticals. The accuracy of any analytical result is contingent upon the integrity of the entire sample preparation chain. Within this chain, the method of sample splitting – or division – is a critical yet often poorly understood step. While seemingly procedural, the choice of splitting method directly influences the validity of downstream measurements and, consequently, operational and economic decisions.<sup>1</sup>

Two mechanical devices dominate the sample division process: the traditional riffle splitter and the rotary splitter. Both aim to produce a representative sub-sample from a larger lot; however, a substantial body of theoretical and applied research demonstrates that their underlying principles – and thus their statistical outcomes – are fundamentally different. In practice, a tension exists between the recognized operational superiority of rotary splitters for routine work and their alleged theoretical incompatibility with the calibration of Gy's formula for the fundamental sampling error (FSE). This tension frames a persistent disconnect between sampling theory and widespread laboratory practice.

The conflict becomes particularly acute in the context of heterogeneity testing, a procedure essential for calibrating the

coefficients  $K$  and  $\alpha$  in Gy's formula. These coefficients are not intrinsically significant but serve as the sole pathway to developing a sampling nomogram or protocol for a given material.<sup>2–5</sup> At the 11th World Conference on Sampling and Blending (WCSB11), practitioners highlighted a methodological dilemma: whether to use riffle or rotary splitting to divide particle size fractions for variance determination.<sup>6</sup> A pointed observation from a leading sampling theorist captured the core issue: 'Good rotary splitting (with carousel) is much superior to riffle splitting, but it simply can't be used for any heterogeneity study aimed at calibrating Gy's formula, because that formula is for a random sample and a rotary split sample is NOT random' (D. François-Bongarçon, personal communication, 22 May 2024). This presents a clear, unresolved gap: the practical efficiency of rotary splitting appears at odds with the theoretical requirement for randomness in FSE calibration.

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The incompatibility stems from first principles. Gy's FSE formula estimates error arising from constitutional heterogeneity – the innate, particle-scale uneven distribution of an analyte.<sup>7</sup> Its derivation assumes a simple random sampling model, wherein every particle has an equal probability of being selected.<sup>8</sup> The riffle splitter, by dividing a falling stream via a series of chutes, subjects each particle to a sequence of Bernoulli trials, approximating this random model.<sup>9</sup> In contrast, the rotary splitter operates systematically, cutting through the entire falling stream to create a mechanically proportional 'miniature' of the whole.<sup>10,11</sup> This process acts as an effective homogenizer, systematically suppressing the very particle-to-particle fluctuations the FSE seeks to quantify. François-Bongarçon's<sup>12</sup> 'Parable of the Doughnut' offers a pertinent metaphor: using a rotary splitter to study heterogeneity is akin to blending a nut-filled doughnut into a uniform paste – the resulting homogeneity reveals nothing about the original size and distribution of the nuts (or ore nuggets). Consequently, employing a rotary splitter for calibration would inherently underestimate the true FSE, invalidating the resulting protocol and potentially leading to dangerously undersized sample masses in full-scale operations.<sup>8,13</sup>

Empirical studies robustly support the use of rotary splitters for routine analytical sample preparation, where the goal is efficient, unbiased mass reduction. Classic work by Allen<sup>14</sup> and Allen and Khan<sup>15</sup> established the 'Golden Rule of Sampling', demonstrating the superiority of devices that sample moving streams, with rotary splitters showing minimal error (0.42%) compared to other methods. This performance is attributed to their ability to mitigate grouping and segregation errors (GSE) through a high frequency of incremental cuts.<sup>16</sup> However, this same 'variance suppression effect' is precisely why the device is deemed unsuitable for heterogeneity calibration – it reflects mechanical averaging, not fundamental material variance. Thus, a clear, context-dependent understanding is required: the riffle splitter remains the indispensable tool for scientific calibration, while the rotary splitter is optimal for routine representativeness. This distinction, implicitly supported by standards such as ISO 11648-2,<sup>17</sup> has yet to be fully elucidated through controlled experiment.

This paper aims to clarify the nuanced but critical distinction between representative sampling (a practical objective) and random sampling (a theoretical prerequisite) within the Theory of Sampling. To move the debate from theoretical contention to empirical evidence, the primary objective is to experimentally validate the variance capture – or suppression – characteristics of a rotary splitter relative to a riffle splitter benchmark. By evaluating performance across a range of split ratios, this study seeks to determine under what operational conditions a rotary splitter can suppress compositional variance and to quantify that effect. The findings are intended to provide evidence-based guidance for practitioners, aligning device selection with specific sampling objectives, whether for protocol calibration or routine analytical preparation.

## Methods and materials

### Experimental rationale and design

The experimental work was designed to evaluate the performance of a Rocklabs Boyd Elite rotating sample divider

(RSD) in two distinct yet related domains: (1) its mass-based mechanical accuracy under controlled conditions, and (2) its compositional representativeness in terms of its ability to mitigate GSE. Performance was benchmarked against two critical baselines: the intrinsic heterogeneity of the test lot, and the conventional standard for representative splitting, a 20-vane riffle splitter.

### Equipment and splitting conditions

The Rocklabs Boyd Elite RSD (Figure 1(a)) features an adjustable splitter ring capable of proportional splitting from 0% to 25% (Figure 1(b)). The device was tested at five target split ratios: 5%, 10%, 15%, 20% and 25%. Each setting was replicated 16 times to ensure statistical robustness and to capture operational variability.

### Test material and tracer additives

A homogeneous lot of crushed basalt (6400 g, 73% passing 2 mm) was seeded with four tracer additives spanning a wide range of densities to serve as sensitive indicators of segregation. The additives, quartz (rock chips), steel balls, lead balls and tungsten carbide chips, were introduced in equal numbers (160 pieces each) but varied substantially in individual mass and density (Figure 2(a)). Their physical characteristics are summarized in Table 1. The total mass of additives was 626 g, yielding a final test lot mass of approximately 7026 g for all experiments (Figure 2(b)).

### Experimental procedure

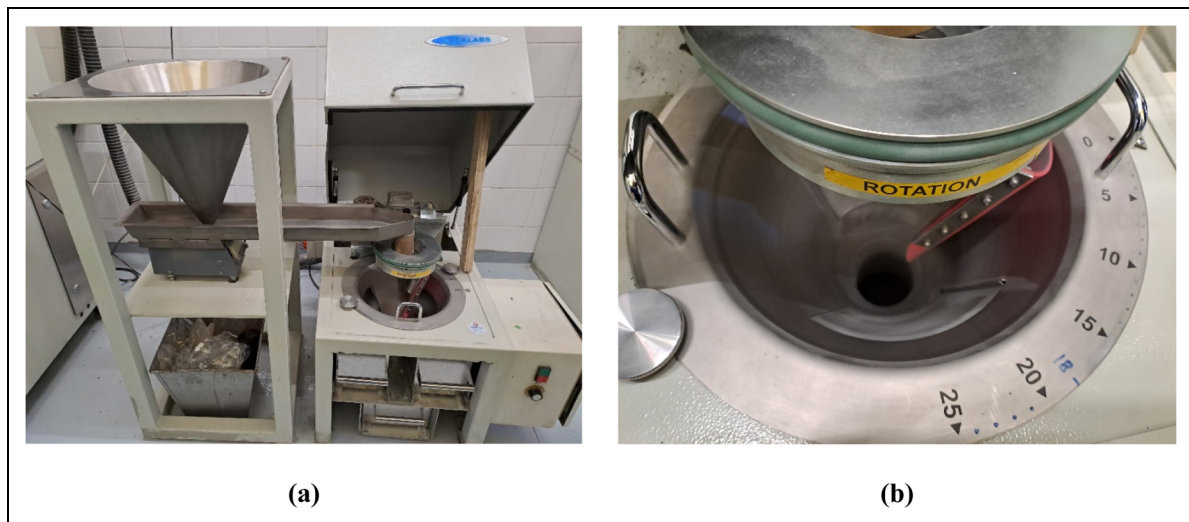
Results for all the splitting data for 5%, 10%, 15%, 20% and 25% splits to the left-hand side (LHS) and right-hand side (RHS) jars on the Boyd Elite Rotary Sample Divider are provided in Appendix A.

- (a) Baseline 1: Intrinsic Heterogeneity (GSE Baseline)
 

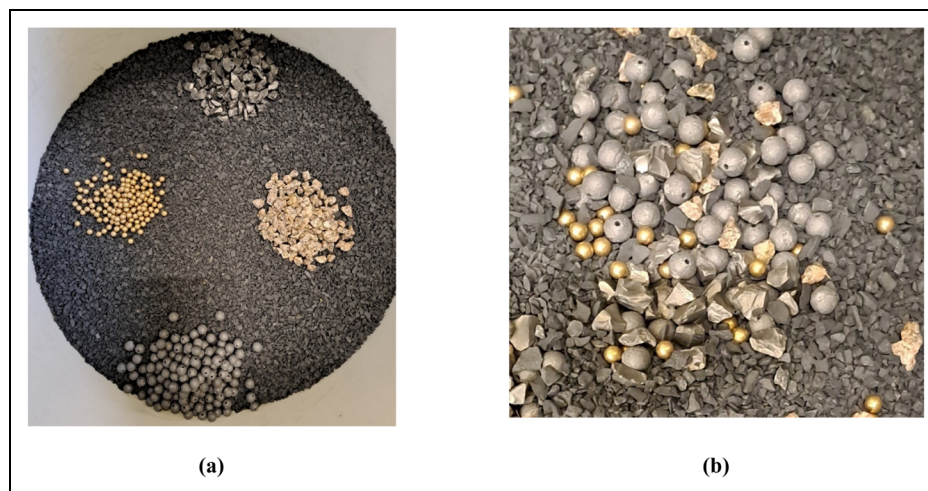
With the RSD set to 0% split, 16 consecutive samples were collected directly from the vibratory feeder outlet. This procedure was repeated three times to quantify the inherent compositional variability of the test lot – representing the maximum GSE under uncontrolled conditions.
- (b) Baseline 2: Riffle Splitter Benchmark
 

The same homogenized lot was processed using the 20-vane riffle splitter to establish the conventional benchmark for variance reduction. Sixteen subsamples were collected, and tracer counts were recorded to determine mean counts, variance and coefficient of variation (CV).
- (c) RSD Performance Testing

The RSD was operated at each of the five target split ratios (5%, 10%, 15%, 20% and 25%). For each ratio, 16 replicate splits were performed. Both the mass delivered to the left (LHS) and right (RHS) collectors and the centre (unsplit) fraction were weighed to assess mechanical bias. The collected subsamples were then manually sorted to count each tracer type, enabling calculation of compositional precision and representativeness.



**Figure 1.** Components of the Rocklabs Boyd Elite Sample Divider: (a) Rotating sample divider (RSD). (b) Adjustable splitter ring with settings from 0% to 25%.



**Figure 2.** Introduction and homogenization of density tracer additives into the crushed basalt test lot. (a) 160 fragments each of the rock chips (RHS), tungsten carbide (top), steel balls (LHS), and lead balls (bottom) prior to mixing. (b) Additives being returned to the crushed basalt prior to homogenization.

### Data and statistical analysis

Mass-based performance was evaluated by comparing target versus achieved split percentages, with particular attention to directional (LHS/RHS) bias and spillage. Compositional performance was assessed through the variance and CV of tracer counts across replicates.

Statistical comparisons employed Welch's analysis of variance (ANOVA)<sup>18</sup> and Kruskal–Wallis tests<sup>19</sup> to account for heteroscedasticity and non-normality. Post-hoc comparisons used the Games–Howell procedure.<sup>20</sup> Effect sizes ( $\omega^2$ ) were calculated to partition variance contributions between splitting method, split ratio and tracer density. All analyses were conducted at a significance level of  $\alpha = 0.05$ .

## Results

### Mass-based mechanical performance of the rotary splitter

Data from 16 replicate splits at each of the five target ratios (5%, 10%, 15%, 20% and 25%) are summarized in Table 2. The results reveal a consistent operational profile characterized by a

systematic positive bias, wherein the total mass delivered to the side collectors (LHS + RHS) exceeded the target specification. Notably, this bias was asymmetric between the two collectors.

The main findings from the mass-balance data are:

- Asymmetric Bias:** The LHS collector consistently received a disproportionately larger share of the excess mass relative to the RHS, indicating a directional bias inherent to the mechanical design (Figure 3).
- Bias Magnitude:** The LHS bias was substantial, ranging from +0.50 to +1.70 percentage points across split ratios, whereas the RHS bias was markedly smaller (−0.16 to +0.61 points).
- Controlled Operation:** Minimal and consistent spillage (<10 g average) confirmed repeatable test conditions. The high 'Centre %' values reflect the unsplit portion of the lot, consistent with the device's design as a proportional divider.

This quantified 'bias map' establishes the mechanical baseline for the device. The critical question for its sampling

**Table 1.** Number, density, size and mass of the four tracer additives.

Additive	Number	Density (g/cm <sup>3</sup> )	Avg. diameter (cm)	Total mass (g)
Rock chips	160	~2.65	>0.475	29.43
Steel balls	160	~7.85	0.43	60.56
Lead balls	160	~11.34	0.7	293.45
Tungsten carbide chips	160	~15.63	0.3-0.7	269.37

utility is whether this mass asymmetry translates into compositional bias when processing segregated materials, which is addressed in the following sections.

### Compositional performance and GSE mitigation

To evaluate the RSD's core function, namely obtaining compositionally representative samples, its ability to suppress variance was compared directly against two benchmarks using the tracer additives.

**Establishing baselines.** Primary Lot Heterogeneity (GSE Baseline): With the RSD set to 0% split, 16 consecutive samples were collected directly from the vibratory feeder. The variable count of high-density tracers in these samples (Appendix B) quantified the severe intrinsic GSE of the test lot. This test was repeated three times; sample mass data are summarized in Tables 3 and 4.

The variability in sample masses from the vibratory feeder is depicted in Figure 4(a), while the average sample mass profile (Figure 4(b)) shows the expected tapering at the start and end of each run, reflecting flow instability. This established the maximum achievable variance against which all subsequent splitting methods were compared.

The observed increase in average mass for mid-stream samples (positions 6–10) indicates that denser additives (steel, lead, tungsten carbide) had segregated toward the centre of the feed hopper and entered the stream as a group. Visual evidence of this variable additive grouping is provided in Figure 5.

**Riffle Splitter Benchmark:** Processing the same lot with a 20-vane riffle splitter established the conventional benchmark for GSE reduction. The riffle splitter significantly reduced variance compared to the vibratory feeder, as summarized in Table 5 and illustrated in Figure 6.

The difference in CVs between the vibratory feeder and riffle splitter is further illustrated in Figure 7, which also

compares sample mass variability across the three riffle-splitter runs.

**RSD performance versus benchmarks.** The compositional precision and representativeness of the RSD at all target splits were calculated from tracer counts (Appendix C) and compared to the established benchmarks (Table 6).

The decisive comparative result is presented in Table 7 and Figure 8.

The RSD achieved better precision and representativeness than the riffle splitter at all split settings except 5%. For splits of 10%, 15%, 20% and 25%, representativeness values were consistently below 10%, outperforming the conventional benchmark. The poorer performance at the 5% split (precision >30%) is attributable to the smaller extracted sample mass (~332 g) and the narrower mechanical aperture at this setting.

**Mechanism for superior performance.** The RSD's advantage is fundamentally mechanistic. During the processing of the 7026 g lot, the rotating divider completes approximately 246 full rotations, each delivering a minute increment to the sample collectors. This represents a greater than twelve-fold increase in the number of incremental cuts (246) compared to a standard riffle splitter (20 vanes). This principle – that a larger number of smaller increments more effectively averages out local segregation – directly explains the RSD's superior suppression of compositional variance and its enhanced mitigation of GSE relative to the riffle splitter.

Appendices A, B and C contain the full raw data for mass splits, vibratory/rifle baseline counts and RSD additive counts, respectively.

## Statistical analysis

### Statistical analysis framework

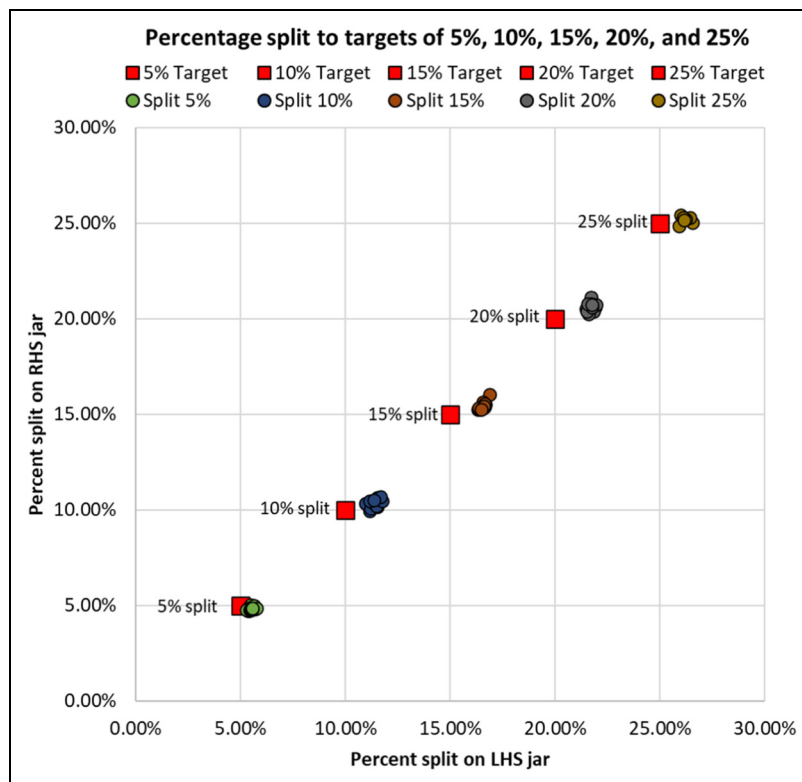
The experimental data were analysed using robust statistical methods to account for significant heteroscedasticity and non-normality. Welch's ANOVA,<sup>18</sup> Kruskal–Wallis tests,<sup>19</sup> and non-parametric factorial analyses<sup>21</sup> were employed. Results are presented in relation to two main experimental factors: splitting method (including split ratio) and tracer density.

### Overall method effect

The choice of splitting method and ratio was the dominant source of variation, accounting for 76–85% of the total variance in particle counts (Welch's  $F(4, 124.8) = 872.45$ ,  $*p < 0.0001$ ;  $\omega^2 = 0.847$ ). Post-hoc comparisons<sup>20</sup> revealed a clear performance hierarchy based on split ratio, as summarized in Table 8.

**Table 2.** Summary of RSD mass-based performance across target split ratios ( $n = 16$  replicates per ratio).

Target split (%)	Avg. LHS %	Avg. RHS %	Avg. total (LHS + RHS) (%)	Split achieved (%)	Avg. centre %	Avg. spillage (g)	Key bias observation
5	5.50	4.84	10.34	5.17	89.43	6.09	LHS bias: +0.50%, RHS bias: -0.16%
10	11.37	10.33	21.70	10.85	78.26	1.57	LHS bias: +1.37%, RHS bias: +0.33%
15	16.50	15.38	31.88	15.94	67.94	1.44	LHS bias: +1.50%, RHS bias: +0.38%
20	21.70	20.61	42.31	21.16	57.64	1.85	LHS bias: +1.70%, RHS bias: +0.61%
25	26.09	25.20	51.29	25.65	48.73	1.50	LHS bias: +1.09%, RHS bias: +0.20%



**Figure 3.** Target versus achieved split mass for the LHS and RHS collectors, illustrating the consistent, asymmetric bias of the RSD.

**Tracer density and interaction effects**

A significant but minor main effect of tracer density was observed ( $\omega^2 = 0.023$ ), with lead particles consistently recovered in higher counts than other tracers ( $*p^* < 0.05$ ). A statistically significant Method  $\times$  Tracer interaction was also detected ( $*p^* = 0.0008$ ); however, it explained only 1.6% of the total variance. This interaction manifested as a more pronounced preferential recovery of dense lead particles at higher rotary split ratios (15–20%).

**Variance comparison: Rotary versus riffle**

The primary metric for sample representativeness, the variance in particle counts between subsamples, depended critically on the split ratio, as visualized in Figure 9.

- (a) At 5% Split: The rotary splitter produced higher variance than the riffle splitter.
- (b) At 10% Split: Variance was comparable between methods.
- (c) At 15–20% Split: The rotary splitter significantly suppressed particle count variance, achieving a 40–50%

**Table 3.** Initial sample mass, vibratory feed rate, sample mass and collection time for GSE baseline sampling.

Sampler characteristics	Measurement
Initial sample mass in feed hopper (g)	7025.95
Total time to clear feed hopper (sec)	357
Feed rate on vibratory feeder (g/sec)	19.68
Sample mass for each of 16 samples (g)	439.12
Time to collect each 439 g sample (sec)	22.31

reduction in the CV compared to the riffle splitter (Brown and Forsythe,<sup>22</sup> Levene’s test,  $p < 0.02^{23}$ ).

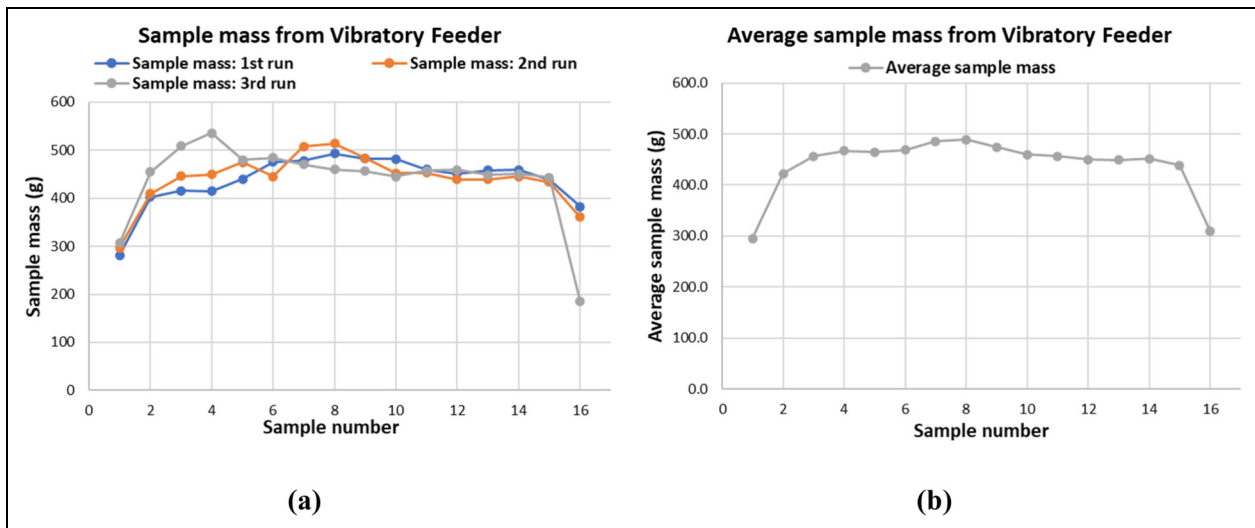
**Discussion**

**The split ratio dictates performance**

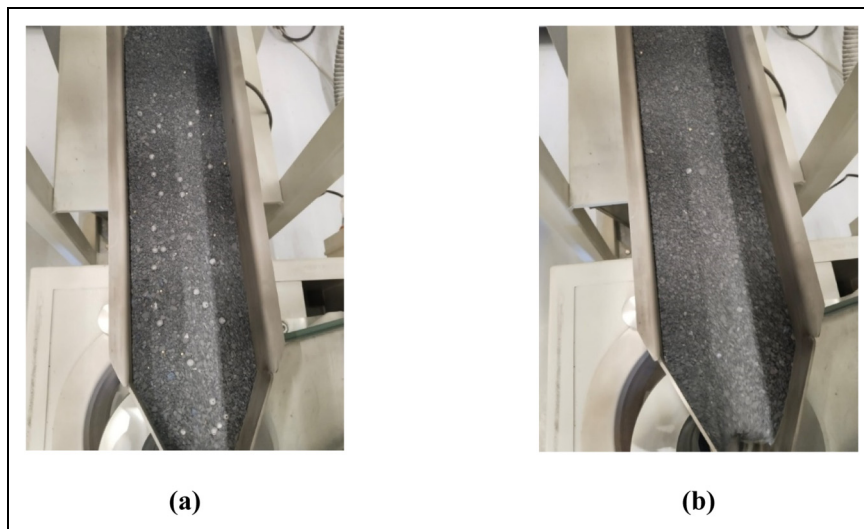
The results reframe the comparison between rotary and riffle splitting. The critical factor is not the mechanism type *per se*, but the effective split ratio and the resulting sample mass it

**Table 4.** Sample masses for three runs of 16 sample each showing average and total sample masses collected.

Sample no.	Sample mass: first run	Sample mass: second run	Sample mass: third run	Average sample mass
1	281.74	297	306.59	295.1
2	402.65	409.91	454.62	422.4
3	415.78	445.68	508.53	456.7
4	414.58	449.61	536.01	466.7
5	439.63	474.64	480.42	464.9
6	475.46	445.35	484.5	468.4
7	478.12	508.19	470.89	485.7
8	492.91	514.12	460.41	489.1
9	482.65	483.49	456.82	474.3
10	481.64	452.18	445.62	459.8
11	459.85	453.35	457.15	456.8
12	451.18	438.95	459.33	449.8
13	458.67	439.34	448.62	448.9
14	458.74	444.82	450.03	451.2
15	438.23	433.75	443.69	438.6
16	382.12	361	184.9	309.3
Total	7013.95	7051.38	7048.13	7037.82



**Figure 4.** Sample mass output from the vibratory feeder used as the baseline for comparison. (a) Sample mass variability for three vibratory feeder runs. (b) Average sample mass profile, showing tapered flow at the beginning and end of each run.



**Figure 5.** Visual evidence of density-based segregation and additive-grouping in the vibratory feeder chute. (a) Numerous steel balls are evident in the stream of crushed basalt. (b) Very few tracer additive in the stream of crushed basalt.

**Table 5.** Mean, variance and coefficient of variation (CV) for high-density additives: comparison between vibratory feeder and riffle splitter.

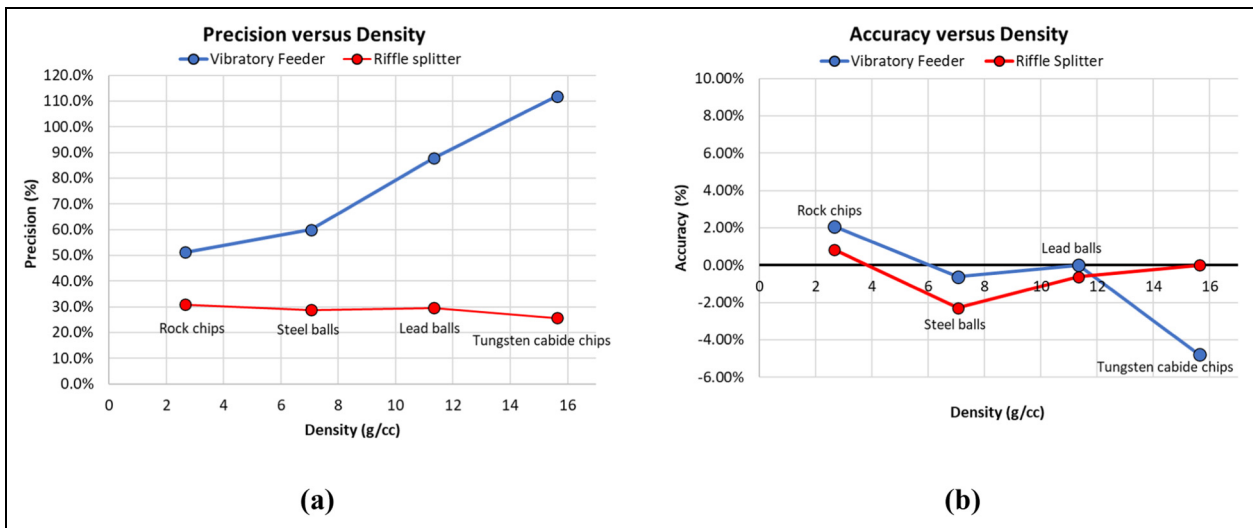
Additive	Vibratory feeder					Riffle splitter				
	Rock	Steel	Lead	TC chips	Mass	Rock	Steel	Lead	TC chips	Mass
Mean	10.21	9.94	10.00	9.52	439.86	10.08	9.77	9.94	10.00	442.44
Variance	27.32	35.42	77.19	113.36	3954.04	9.65	7.93	8.61	6.60	78.86
CV	51.20%	59.89%	87.86%	111.83%	14.30%	30.81%	28.81%	29.53%	25.68%	2.01%
Density	2.65	7.05	11.34	15.63		2.65	7.05	11.34	15.63	

delivers. The rotary splitter does not inherently suppress variance; its performance is contingent on operational parameters. At its lowest setting (5%), it delivers a small sample (~332 g) that remains highly susceptible to the inherent GSE of the heterogeneous lot, resulting in amplified variance. In contrast, at higher settings (15–20%), it extracts a substantially larger sample mass (>1 kg). This larger mass inherently provides better averaging of particulate heterogeneity, which is the fundamental mechanism behind the observed variance

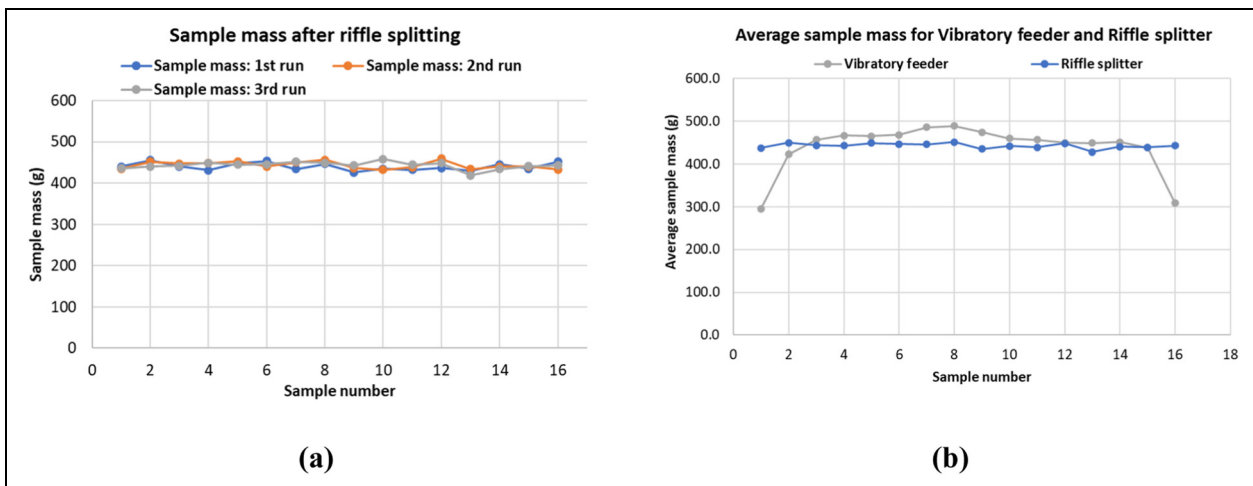
suppression. The riffle splitter, with its fixed geometry, consistently provides a moderate, relatively small sample (~440 g) with excellent mass consistency but higher particle-count variance.

**Mechanism of variance suppression**

The superior performance of the rotary splitter at high split ratios can be attributed to its operational principle. During



**Figure 6.** Comparison of sampling performance between the vibratory feeder and the riffle splitter. (a) Variation in the precision and density of the additives sampled by the vibratory feeder and the riffle splitter. (b) Variations in the accuracy (bias) and density of the additives sampled by the vibratory feeder and the riffle splitter.



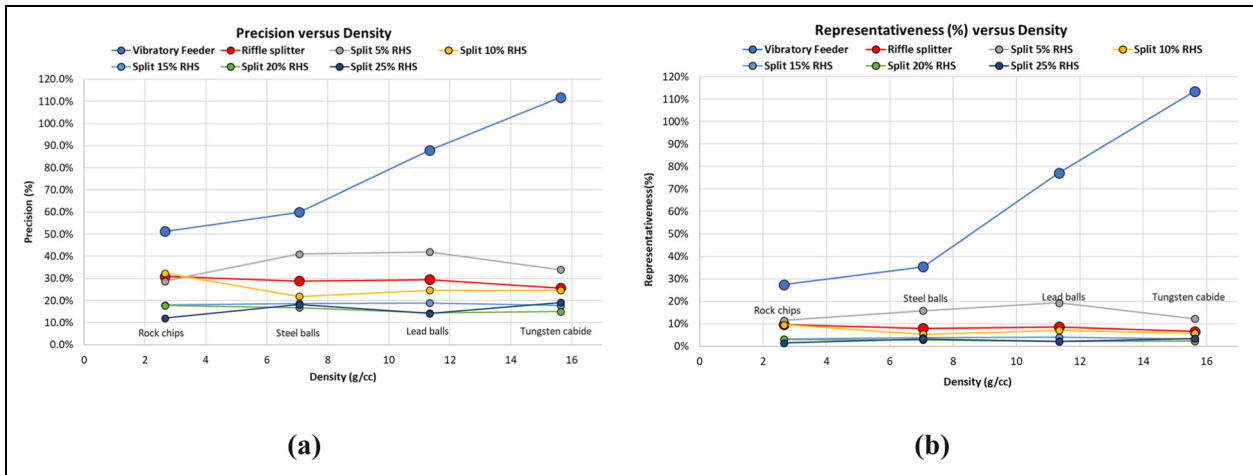
**Figure 7.** Mass variability and performance comparison. (a) Sample mass consistency across three riffle splitter runs. (b) Comparison of average sample masses from vibratory feeder and riffle splitter.

**Table 6.** Precision and accuracy for additives from vibratory feeder and riffle splitter baselines.

	Vibratory feeder					Riffle splitter				
	Rock (%)	Steel (%)	Lead (%)	TC chips (%)	Mass (%)	Rock (%)	Steel (%)	Lead (%)	TC chips (%)	Mass (%)
Precision	51.2	59.9	87.9	111.8	14.3	30.8	28.8	29.5	25.7	2.0
Accuracy	2.08	-0.62	0.00	-4.79	0.20	0.83	-2.29	-0.63	0.00	0.78

**Table 7.** Precision, accuracy and representativeness of four additives at the 5%, 10%, 15%, 20% and 25% splits.

	Precision				Accuracy				Representativeness			
	Rock (%)	Steel (%)	Lead (%)	TC chips (%)	Rock (%)	Steel (%)	Lead (%)	TC chips (%)	Rock (%)	Steel (%)	Lead (%)	TC chips (%)
5% Split	28.71	41.04	41.97	33.92	10.77	-7.28	4.33	2.55	11.48	15.78	19.36	12.17
10% Split	32.36	21.84	24.66	24.60	-6.63	-7.31	5.42	-2.22	9.57	5.39	7.05	5.84
15% Split	17.92	18.72	18.88	17.75	-3.40	-9.72	3.74	-7.15	3.11	3.80	3.98	3.23
20% Split	17.80	16.83	14.36	15.05	-2.34	-6.01	1.88	-5.12	3.08	2.86	2.18	2.30
25% Split	12.06	18.15	14.13	19.10	0.31	0.01	0.98	-1.48	1.48	3.30	2.11	3.52

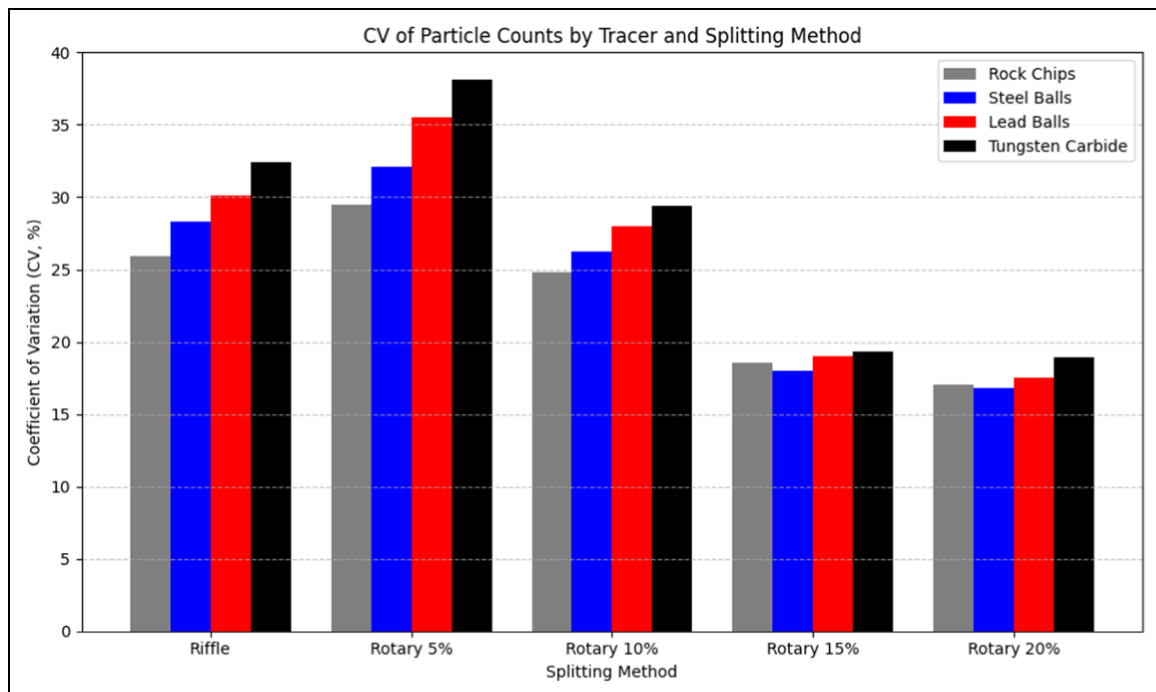


**Figure 8.** Performance comparison of rotary sample divider (RSD) against riffle splitter. (a) Precision versus density for RSD splits and riffle splitter. (b) Representativeness versus density demonstrating RSD superiority for splits >10%.

**Table 8.** Summary of splitting method performance for particle recovery (mean count) and variance (coefficient of variation, CV).

Method	Mean particle count (vs. riffle)	Particle CV (range) (%)	Mass CV (%)	Key statistical comparison versus riffle
Riffle splitter	Reference	25.9–32.4	2.01	–
Rotary 5%	No significant difference ( $p = 0.467$ )	25.9–38.1	6.76	Higher variance for most tracers.
Rotary 10%	+6.58 [5.00, 8.16], $p < 0.0001$	23.7–29.4	5.08	Comparable variance.
Rotary 15%	+15.29 [13.71, 16.87], $p < 0.0001$	17.2–19.3	4.18	Lower particle CV ( $p = 0.016$ ).
Rotary 20%	+20.92 [19.34, 22.50], $p < 0.0001$	16.2–18.9	2.75	Lowest particle CV ( $p = 0.004$ ).

Note: 95% confidence intervals are shown in square brackets for mean differences.



**Figure 9.** Coefficient of variation (CV) for particle counts across tracers and splitting methods. The rotary splitter at 15% and 20% settings demonstrates clear variance suppression relative to the riffle splitter baseline.

the processing of the 7 kg lot, the rotating mechanism collects the sample through approximately 246 incremental cuts. This high frequency of incremental sampling provides more opportunities to average out local segregation than the 20 discrete increments of a standard riffle splitter.

When combined with a larger extracted sample mass, this leads to a more robust mitigation of GSE and lower final sub-sample variance. This mechanistic advantage aligns with the ‘composite sampling’ principle, wherein a greater number of smaller increments enhances representativeness.<sup>16</sup>

### Practical implications for sample preparation

The choice between splitters should be guided by the sampling objective. For maximum mass consistency in small samples, the riffle splitter remains the optimal tool (mass CV = 2.01%). However, for superior compositional representativeness (lower particle-count variance) and the ability to process larger samples, the rotary splitter operated at a 15–20% split ratio is demonstrably superior.

The significant but small interaction effect (Method × Tracer) suggests a slight density-dependent bias in the rotary splitter at high throughputs, which warrants consideration when processing materials with extreme density contrasts. Nevertheless, this effect explained only 1.6% of the total variance and does not detract from the overall performance advantage at appropriate split ratios.

### Reconciling practical and theoretical roles

These findings underscore the contextual correctness of each device. While the rotary splitter is unsuitable for heterogeneity tests aimed at calibrating Gy's FSE formula – due to its systematic, non-random mechanism – it is a highly effective tool for routine sample preparation where the goal is efficient, representative mass reduction for chemical analysis. The riffle splitter, conversely, remains indispensable for scientific calibration where the aim is to quantify fundamental material heterogeneity. This distinction is not about which device is 'better' in an absolute sense, but about aligning tool selection with the specific sampling objective, as implicitly supported by international standards such as ISO 1648-2.<sup>17</sup>

### Conclusions

This study provides a definitive, ratio-dependent resolution to the long-standing practical and theoretical debate regarding variance suppression between rotary and riffle splitting methods. Through systematic experimentation and robust statistical analysis, the following key conclusions are drawn:

- (a) Rotary splitting suppresses subsample variance compared to riffle splitting, but only when operated at sufficiently high split ratios ( $\geq 15\%$ ). At these settings, the rotary splitter reduces particle-count variability by 40–50% relative to the conventional riffle splitter benchmark. At lower split ratios (e.g. 5%), the rotary splitter performs worse or comparably, demonstrating that its variance-reduction capability is conditional, not inherent.
- (b) The dominant factor governing sample representativeness is the split ratio – and thus the extracted sample mass, not the splitting mechanism per se. The split ratio accounted for approximately 85% of the total variance in particle recovery, underscoring that larger sample masses inherently enable better averaging of particulate heterogeneity and more effective mitigation of GSE.
- (c) The mechanistic advantage of the rotary splitter lies in its high frequency of incremental sampling. During processing, the rotary divider executes approximately 246 incremental cuts, compared to

only 20 discrete increments in a standard riffle splitter. This greater number of smaller increments, combined with the ability to deliver larger sample masses at higher split settings, provides a more robust means of averaging out local segregation, leading to superior compositional representativeness.


- (d) Contextual tool selection is critical. The rotary splitter is recommended as a superior tool for routine sample preparation where the objective is efficient, representative mass reduction for chemical analysis, provided it is operated at an appropriate split ratio (15–20%). In contrast, the riffle splitter remains indispensable for scientific calibration of sampling protocols (e.g. for Gy's FSE formula), where the requirement for random particle selection cannot be compromised.

Therefore, the choice between rotary and riffle splitting should be guided not by a generic preference for one device, but by a clear understanding of the sampling objective, whether it is routine analytical representativeness or fundamental heterogeneity characterization. This evidence-based clarification aligns practical sampling operations with theoretical rigor and supports more reliable resource estimation, process control and quality assurance in particulate-dependent industries. Statistical analysis confirms that rotary splitting at  $\geq 15\%$  split ratio significantly suppresses particle-count variance compared to riffle splitting (Welch's  $F = 872.45$ ,  $p < 0.0001$ ; CV reduction 40–50%, post-hoc  $p \leq 0.016$ ). The effect is attributable to the rotary mechanism's higher increment count ( $\approx 246$  vs. 20), which enhances averaging of local segregation. Thus, while riffle splitting remains necessary for FSE calibration, rotary splitting is statistically superior for routine representative sample preparation.

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## Appendix A: Splitting data for 5%, 10% and 15% splits to the LHS and RHS jars on the Boyd Elite Rotary Sample Divider

160 rock chips	29.43
160 steel balls	60.56
160 lead balls	293.45
160 tungsten carbide chips	269.37
Total additives	652.81
Total: crushed rock + additives	7052.81

	5% target Using 6400 g plus additives	Splitting time (sec)		366.21			Split 5%			RHS (%)
		LHS jars	Centre	RHS jars	Total sampled	Total (g)	Spillage (g)	LHS (%)	Centre (%)	
1	7052.81	386.71	6298.03	341.21	727.92	7025.95	26.86	5.48	89.30	4.84
2	7025.95	375.13	6307.84	333.94	709.07	7016.91	9.04	5.34	89.78	4.75
3	7017.55	379.43	6306.91	331.95	711.38	7018.29	-0.74	5.41	89.87	4.73
4	7018.29	384.83	6263.24	353.6	738.43	7001.67	16.62	5.48	89.24	5.04
5	7026.53	375.19	6303.73	340.76	715.95	7019.68	6.85	5.34	89.71	4.85
6	7017.79	370.04	6312.53	335.73	705.77	7018.3	-0.51	5.27	89.95	4.78
7	7017.79	381	6291.47	336.68	717.68	7009.15	8.64	5.43	89.65	4.80
8	7008.37	393.83	6260.43	349.53	743.36	7003.79	4.58	5.62	89.33	4.99
9	7025.68	398.75	6275.62	344.93	743.68	7019.3	6.38	5.68	89.32	4.91
10	7019.23	396.91	6276.97	338.94	735.85	7012.82	6.41	5.65	89.43	4.83
11	7012.39	383.7	6283.44	337.98	721.68	7005.12	7.27	5.47	89.60	4.82
12	7025.75	386.15	6296.78	337.38	723.53	7020.31	5.44	5.50	89.62	4.80
13	7020.09	396.85	6264.31	351.93	748.78	7013.09	7	5.65	89.23	5.01
14	7012.56	402.71	6262.38	342.13	744.84	7007.22	5.34	5.74	89.30	4.88
15	7026.13	390.21	6292.13	337.62	727.83	7019.96	6.17	5.55	89.55	4.81
16	7018.86	388.97	6284.09	341.6	730.57	7014.66	4.2	5.54	89.53	4.87

	10% target Using 6400 g plus additives	Splitting time (sec)		366.21			Split 10%			RHS (%)
		LHS jars	Centre	RHS jars	Total sampled	Total (g)	Spillage (g)	LHS (%)	Centre (%)	
1	7026.12	808.37	5504.46	713.92	1522.29	7026.75	-0.63	11.51	78.34	10.16
2	7025.7	782.58	5539.31	700.38	1482.96	7022.27	3.43	11.14	78.84	9.97
3	7021.55	795.5	5506.03	720.28	1515.78	7021.81	-0.26	11.33	78.42	10.26
4	7021.74	770.33	5521.17	727.46	1497.79	7018.96	2.78	10.97	78.63	10.36
5	7018.49	796.93	5494.67	723.48	1520.41	7015.08	3.41	11.35	78.29	10.31
6	7014.83	792.9	5505.5	714.63	1507.53	7013.03	1.8	11.30	78.48	10.19
7	7012.83	783.89	5518.59	706.24	1490.13	7008.72	4.11	11.18	78.69	10.07
8	7025.22	798.27	5498.39	728.48	1526.75	7025.14	0.08	11.36	78.27	10.37
9	7025.02	803.73	5493.55	725.75	1529.48	7023.03	1.99	11.44	78.20	10.33
10	7022.52	798.14	5495.71	726.32	1524.46	7020.17	2.35	11.37	78.26	10.34
11	7018.99	806.04	5495.59	716.36	1522.4	7017.99	1	11.48	78.30	10.21
12	7018.83	784.04	5499.43	735.46	1519.5	7018.93	-0.1	11.17	78.35	10.48
13	7018.61	825.91	5460.12	734.01	1559.92	7020.04	-1.43	11.77	77.79	10.46
14	7023.18	807.87	5466.06	747.88	1555.75	7021.81	1.37	11.50	77.83	10.65
15	7021.18	819.45	5448.63	751.27	1570.72	7019.35	1.83	11.67	77.60	10.70
16	7018.91	798.36	5481.36	737.91	1536.27	7017.63	1.28	11.37	78.09	10.51

## Appendix A: Splitting data for 20% and 25% splits to the LHS and RHS jars on the Boyd Elite Rotary Sample Divider

	15% target Using 6400 g plus additives	Splitting time (sec)		366.21			Split 15%			RHS (%)
		LHS jars	Centre	RHS jars	Total sampled	Total (g)	Spillage (g)	LHS (%)	Centre (%)	
1	7052.79	1171.95	4790.18	1087.22	2259.17	7049.35	3.44	16.62	67.92	15.42
2	7042.12	1151.99	4801.7	1093.29	2245.28	7046.98	-4.86	16.36	68.19	15.53
3	7046.98	1168.88	4788.37	1088.59	2257.47	7045.84	1.14	16.59	67.95	15.45
4	7046.12	1149.69	4818.61	1076.34	2226.03	7044.64	1.48	16.32	68.39	15.28
5	7044.28	1158.27	4800.53	1084.35	2242.62	7043.15	1.13	16.44	68.15	15.39
6	7043.25	1166.05	4773.34	1101.05	2267.1	7040.44	2.81	16.56	67.77	15.63
7	7040.28	1158.27	4805.03	1077.04	2235.31	7040.34	-0.06	16.45	68.25	15.30
8	7040.22	1166.42	4789.29	1083.11	2249.53	7038.82	1.4	16.57	68.03	15.38
9	7038.93	1153.24	4811.38	1073.07	2226.31	7037.69	1.24	16.38	68.35	15.24
10	7037.55	1171.41	4781.12	1081.58	2252.99	7034.11	3.44	16.65	67.94	15.37
11	7033.98	1172.31	4771.53	1090.79	2263.1	7034.63	-0.65	16.67	67.84	15.51
12	7033.68	1170.28	4766.69	1095.51	2265.79	7032.48	0.19	16.64	67.78	15.58
13	7032.67	1166.75	4784.53	1080.09	2246.84	7031.37	1.74	16.59	68.03	15.36
14	7033.11	1148.72	4801.05	1078.81	2227.53	7028.58	-0.1	16.34	68.31	15.35
15	7028.48	1167.57	4779.55	1083.2	2250.77	7030.32	-0.56	16.61	67.99	15.41
16	7029.76	1156.95	4796.08	1072.72	2229.67	7025.75	4.01	16.46	68.23	15.26

	20% target Using 6400 g plus additives	Splitting time (sec)		366.21				LHS (%)	Split 20%	
		LHS jars	Centre	RHS jars	Total sampled	Total (g)	Spillage (g)		Centre (%)	RHS (%)
1	7050.64	1530.15	4031.22	1490.84	3020.99	7052.21	-1.57	21.70	57.18	21.14
2	7051.21	1527.87	4059.03	1458.23	2986.1	7045.13	6.08	21.67	57.57	20.68
3	7044.44	1513.57	4082.28	1445.19	2958.76	7041.04	3.4	21.49	57.95	20.52
4	7039.81	1537.46	4047.75	1457.2	2994.66	7042.41	-2.6	21.84	57.50	20.70
5	7043.08	1538.55	4053.69	1448.05	2986.6	7040.29	2.79	21.84	57.56	20.56
6	7039.53	1528.54	4057.87	1451.08	2979.62	7037.49	2.04	21.71	57.64	20.61
7	7035.67	1541.52	4045.06	1448.47	2989.99	7035.05	0.62	21.91	57.49	20.59
8	7036.01	1536.48	4065.27	1432.78	2969.26	7034.53	1.48	21.84	57.78	20.36
9	7034.36	1544.76	4031.19	1457.1	3001.86	7033.05	1.31	21.96	57.31	20.71
10	7031.79	1517.21	4089.78	1424.19	2941.4	7031.18	0.61	21.58	58.16	20.25
11	7030.4	1525.63	4051.06	1451.75	2977.38	7028.44	1.96	21.70	57.62	20.65
12	7027.52	1512.41	4082.47	1431.63	2944.04	7026.51	-1.36	21.52	58.09	20.38
13	7025.15	1526.26	4038.86	1458.53	2984.79	7023.65	1.16	21.73	57.49	20.76
14	7024.81	1529.21	4045.73	1448.33	2977.54	7023.27	-0.1	21.77	57.59	20.62
15	7023.17	1512.39	4051.25	1458.85	2971.24	7022.49	-0.1	21.53	57.68	20.77
16	7022.39	1526.51	4032.71	1456.02	2982.53	7015.24	7.15	21.74	57.43	20.73

	25% target Using 6400 g plus additives	Splitting time (sec)		366.21				LHS (%)	Split 25%	
		LHS jars	Centre	RHS jars	Total sampled	Total (g)	Spillage (g)		Centre (%)	RHS (%)
1	7050.15	1841.11	3434.71	1776.23	3617.34	7052.05	-1.9	26.11	48.72	25.19
2	7050.81	1872.83	3412.87	1764.1	3636.93	7049.8	1.01	26.56	48.40	25.02
3	7050.36	1864.97	3434.12	1784.3	3649.27	7083.39	-33.03	26.45	48.71	25.31
4	7046.53	1831.07	3429.64	1790.85	3621.92	7051.56	-5.03	25.99	48.67	25.41
5	7050.69	1826.86	3470.58	1751.52	3578.38	7048.96	1.73	25.91	49.22	24.84
6	7049.26	1850.11	3419.69	1777.84	3627.95	7047.64	1.62	26.25	48.51	25.22
7	7046.36	1839.84	3421.16	1783.53	3623.37	7044.53	1.83	26.11	48.55	25.31
8	7043.78	1842.28	3424.84	1772.91	3615.19	7040.03	3.75	26.15	48.62	25.17

### Appendix B: Data for three runs (first run, second run and third run) for 16 samples collected from the vibratory feeder and the 20 vane riffle splitter

Vibratory feeder							Riffle splitter						
Run	Count	Rock	Steel	Lead	TC chips	Mass (g)	Count	Rock	Steel	Lead	TC chips	Mass	
First run	1	20	0	0	0	281.74	First run	1	13	15	11	9	440.57
	2	10	1	0	0	402.65		2	11	10	16	12	455.9
	3	11	6	2	0	415.78		3	9	6	8	13	440.56
	4	13	10	6	0	414.58		4	12	7	4	7	431.66
	5	7	4	15	13	439.63		5	5	12	11	10	447.66
	6	5	18	18	2	475.46		6	13	13	9	11	454.22
	7	4	14	25	23	478.12		7	6	10	10	6	433.8
	8	10	18	14	31	492.91		8	8	6	15	5	446.41
	9	7	15	14	22	482.65		9	12	14	11	9	425.88
	10	12	20	14	16	481.64		10	8	6	11	10	434.93
	11	9	14	12	11	459.85		11	11	6	12	13	432.03
	12	5	8	10	8	451.18		12	13	6	8	9	436.99
	13	14	10	14	3	458.67		13	7	15	10	6	430.5
	14	13	11	16	5	458.74		14	13	14	8	12	446.11
	15	10	7	1	1	438.23		15	7	6	5	15	435.18

(continued)

Continued.

Vibratory feeder							Riffle splitter						
Run	Count	Rock	Steel	Lead	TC chips	Mass (g)	Run	Count	Rock	Steel	Lead	TC chips	Mass
	16	12	4	0	0	382.12		16	14	9	11	11	451.96
Second run	1	7	0	0	0	297	Second run	1	16	6	8	8	435.42
	2	6	3	2	2	409.91		2	7	10	10	16	451.83
	3	4	16	25	6	445.68		3	11	10	10	11	447.65
	4	6	14	13	11	449.61		4	13	11	11	11	448.17
	5	13	11	18	20	474.64		5	11	8	14	11	452.93
	6	7	17	11	10	445.35		6	10	11	11	8	440.9
	7	12	15	31	40	508.19		7	12	7	10	10	450.44
	8	7	17	23	29	514.12		8	12	13	9	13	456.55
	9	6	10	11	18	483.49		9	4	11	15	5	436.88
	10	6	15	8	7	452.18		10	11	6	5	8	431.98
	11	7	10	8	7	453.35		11	14	12	5	9	438.51
	12	7	17	6	4	438.95		12	7	9	15	11	459.26
	13	15	5	2	6	439.34		13	11	9	9	12	434.43
	14	25	6	1	1	444.82		14	6	12	9	11	440.24
	15	27	2	0	0	433.75		15	4	10	12	9	440.06
	Third run	16	8	1	0	0		361	Third run	16	12	12	7
1		8	10	5	2	306.59	1	10		7	9	6	435.72
2		5	21	27	21	454.62	2	6		12	5	11	440.36
3		6	16	27	36	508.53	3	8		9	11	7	442.52
4		7	20	26	33	536.01	4	6		9	8	10	449.61
5		16	12	13	9	480.42	5	15		10	10	14	445.34
6		7	8	16	15	484.5	6	9		8	8	12	445.77
7		8	11	5	8	470.89	7	9		14	15	9	452.56
8		13	6	7	6	460.41	8	12		8	12	9	449.84
9		16	7	7	8	456.82	9	7		7	10	14	442.77
10		11	12	6	7	445.62	10	16		12	12	13	458.75
11		8	13	11	7	457.15	11	12		10	11	10	445.21
12		11	7	5	9	459.33	12	10		10	12	10	448.27
13		13	9	4	0	448.62	13	7		15	3	8	418.88
14		21	3	0	0	450.03	14	10		11	10	7	434.23
15		12	3	1	0	443.69	15	14		10	10	9	441.13
16	3	0	0	0	184.9	16	10	5	11	12	443.11		
Mean	10.21	9.94	10.00	9.52	439.86	Mean	10.08	9.77	9.94	10.00	442.44		
Variance	27.32	35.42	77.19	113.36	3954.04	Variance	9.65	7.93	8.61	6.60	78.86		
CV	51.2%	59.9%	87.9%	111.8%	14.3%	CV	30.8%	28.8%	29.5%	25.7%	2.0%		
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63			

**Appendix C: Data for 5%, 10%, 15%, 20% and 25% splits to the LHS and RHS sample containers on the Boyd RSD**

Split 5% LHS						Split 5% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
1	8	11	16	9	386.71	1	6	7	5	9	341.21
2	8	4	14	4	375.13	2	9	9	4	7	333.94
3	10	6	12	8	379.43	3	11	7	4	9	331.95
4	4	4	9	12	384.83	4	13	7	10	12	353.6
5	7	11	6	7	375.19	5	11	5	11	6	340.76
6	13	6	6	9	370.04	6	10	6	3	7	335.73
7	11	7	9	8	381	7	9	5	7	7	336.68
8	7	12	13	12	393.83	8	10	12	8	12	349.53
9	10	7	16	11	398.75	9	8	9	5	12	344.93
10	12	6	14	11	396.91	10	13	8	7	5	338.94

(continued)

Continued.

Split 5% LHS						Split 5% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
11	14	7	15	4	383.7	11	7	5	5	8	337.98
12	8	12	9	12	386.15	12	6	9	9	4	337.38
13	9	3	11	16	396.85	13	13	17	7	10	351.93
14	9	6	14	15	402.71	14	5	6	9	8	342.13
15	12	5	9	12	390.21	15	6	7	6	7	337.62
16	7	5	15	10	388.97	16	7	13	6	6	341.6
Mean	9.31	7.00	11.75	10.00	386.90	Mean	9.00	8.25	6.63	8.06	340.99
Variance	6.90	8.53	11.40	11.33	87.90	Variance	7.33	10.87	5.32	6.06	38.95
CV	0.28	0.42	0.29	0.34	0.02	CV	0.30	0.40	0.35	0.31	0.02
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63	

Split 10% LHS						Split 10% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
1	13	19	27	21	808.37	1	13	21	20	10	713.92
2	21	19	19	15	782.58	2	0	14	14	14	700.38
3	22	12	28	17	795.5	3	15	11	19	16	720.28
4	14	13	16	13	770.33	4	11	17	18	19	727.46
5	9	15	21	23	796.93	5	17	26	18	11	723.48
6	30	21	24	17	792.9	6	15	9	15	14	714.63
7	17	20	21	16	783.89	7	18	16	14	10	706.24
8	17	17	20	20	798.27	8	17	17	18	19	728.48
9	20	13	26	22	803.73	9	18	21	19	11	725.75
10	22	16	16	27	798.14	10	17	16	17	17	726.32
11	20	23	22	24	806.04	11	19	16	10	16	716.36
12	24	16	12	20	784.04	12	23	17	18	18	735.46
13	19	22	32	18	825.91	13	11	14	14	21	734.01
14	19	22	22	22	807.87	14	25	16	20	21	747.88
15	21	18	26	22	819.45	15	15	21	18	25	751.27
16	18	17	18	22	798.36	16	10	15	19	15	737.91
Mean	19.13	17.69	21.88	19.94	798.27	Mean	15.25	16.69	16.94	16.06	725.61
Variance	22.92	11.70	26.65	13.53	198.09	Variance	33.00	16.90	7.66	19.00	191.53
CV	0.25	0.19	0.24	0.18	0.02	CV	0.38	0.25	0.16	0.27	0.02
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63	

Split 15% LHS						Split 15% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
1	38	31	32	32	1171.95	1	22	23	24	21	1087.22
2	21	25	24	27	1151.99	2	29	27	29	24	1093.29
3	33	19	40	26	1168.88	3	19	30	25	24	1088.59
4	29	21	34	17	1149.69	4	26	31	18	24	1076.34
5	29	28	22	27	1158.27	5	20	21	30	20	1084.35
6	23	25	35	24	1166.05	6	33	30	29	27	1101.05
7	24	20	31	28	1158.27	7	29	21	24	20	1077.04
8	20	20	32	26	1166.42	8	27	19	25	22	1083.11
9	24	23	28	25	1153.24	9	30	21	20	21	1073.07
10	23	29	30	35	1171.41	10	35	14	20	27	1081.58
11	26	25	35	23	1172.31	11	23	23	30	18	1090.79

(continued)

Continued.

Split 15% LHS						Split 15% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
12	27	33	31	27	1170.28	12	23	21	25	30	1095.51
13	24	23	35	23	1166.75	13	26	27	26	21	1080.09
14	26	28	25	27	1148.72	14	23	19	23	22	1078.81
15	22	22	29	36	1167.57	15	19	29	22	23	1083.2
16	27	20	31	26	1156.95	16	25	23	22	20	1072.72
Mean	26.00	24.50	30.88	26.81	1162.42	Mean	25.56	23.69	24.50	22.75	1084.17
Variance	21.33	18.27	21.32	21.23	70.31	Variance	22.66	23.43	13.47	9.93	66.00
CV	0.18	0.17	0.15	0.17	0.01	CV	0.19	0.20	0.15	0.14	0.01
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63	

Split 20% LHS						Split 20% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
1	28	36	40	30	1530.15	1	44	30	43	43	1490.84
2	27	31	36	34	1527.87	2	35	32	32	34	1458.23
3	40	38	34	26	1513.57	3	34	30	36	28	1445.19
4	29	38	29	40	1537.46	4	35	26	41	31	1457.2
5	34	36	38	38	1538.55	5	37	25	34	33	1448.05
6	30	25	36	38	1528.54	6	40	40	35	28	1451.08
7	38	49	45	33	1541.52	7	38	31	28	34	1448.47
8	18	35	40	39	1536.48	8	34	31	26	30	1432.78
9	37	36	36	43	1544.76	9	35	36	34	34	1457.1
10	43	34	39	33	1517.21	10	31	32	23	28	1424.19
11	36	37	34	34	1525.63	11	32	26	37	29	1451.75
12	30	33	31	34	1512.41	12	22	29	29	28	1431.63
13	26	32	35	36	1526.26	13	38	29	39	33	1458.53
14	36	31	32	39	1529.21	14	37	22	33	31	1448.33
15	47	42	37	24	1512.39	15	31	32	42	28	1458.85
16	34	28	40	39	1526.51	16	33	36	42	26	1456.02
Mean	33.31	35.06	36.38	35.00	1528.03	Mean	34.75	30.44	34.63	31.13	1451.14
Variance	51.56	31.00	15.98	26.27	104.33	Variance	23.13	20.40	36.12	16.92	222.72
CV	0.22	0.16	0.11	0.15	0.01	CV	0.14	0.15	0.17	0.13	0.01
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63	

Split 25% LHS						Split 25% RHS					
Count	Rock	Steel	Lead	TC chips	Mass	Count	Rock	Steel	Lead	TC chips	Mass
1	41	43	37	45	1841.11	1	43	41	55	30	1776.23
2	42	47	42	57	1872.83	2	45	35	37	36	1764.1
3	50	30	40	59	1864.97	3	39	51	50	33	1784.3
4	44	40	44	38	1831.07	4	46	37	35	41	1790.85
5	48	27	44	37	1826.86	5	34	51	33	34	1751.52
6	43	51	47	40	1850.11	6	40	35	51	39	1777.84
7	37	44	44	39	1839.84	7	47	41	43	44	1783.53
8	45	49	44	41	1842.28	8	31	49	38	38	1772.91
Mean	43.75	41.375	42.75	44.5	1846.13	Mean	40.625	42.5	42.75	36.875	1775.16
Variance	16.5	75.70	9.36	75.43	251.63	Variance	33.41	47.71	68.79	20.70	156.33
CV	0.09	0.21	0.07	0.20	0.01	CV	0.14	0.16	0.19	0.12	0.01
Density	2.65	7.05	11.34	15.63		Density	2.65	7.05	11.34	15.63	